Lecture 3. The Compton Effect. De Broglie Waves. The Theory of Wave-Particle Duality

Scattering – рассеяние

Goal, To describe the Compton Effect and its relation to the theory of wave-particle duality

The Compton Effect

In 1919, Einstein concluded that a photon of energy E travels in a single direction (unlike a spherical wave) and carries a momentum equal to E/c or hf/c. In his own words, “if a bundle of radiation causes a molecule to emit or absorb an energy packet hf, then momentum of quantity hf/c is transferred to the molecule, directed along the line of the bundle for absorption and opposite the bundle for emission.” In 1923, Arthur Holly Compton (1892-1962) and Peter Debye independently carried Einstein’s idea of photon momentum farther. They realized that the scattering of X-ray photons from electrons could be explained by treating photons as point-like particles with energy hf and momentum hf/c, and by conserving energy and momentum of the photon-electron pair in a collision.

Prior to 1922, Compton and his coworkers had accumulated evidence that showed that classical wave theory failed to explain the scattering of x-rays from electrons. According to classical wave theory, incident electromagnetic waves of frequency f_0 should accelerate electrons, forcing them to oscillate and reradiate at a frequency $f < f_0$ as in figure 3.1a. Furthermore, according to classical theory, the frequency or wavelength of the scattered radiation should depend on the time of exposure of the sample to the incident radiation as well as the intensity of the incident radiation. Contrary to these predictions, Compton’s experimental results showed that the wavelength shift of X-rays scattered at a given angle depends on only on the scattering angle. Figure 3.1b shows the quantum picture of the transfer of momentum and energy between an individual x-ray photon and an electron.

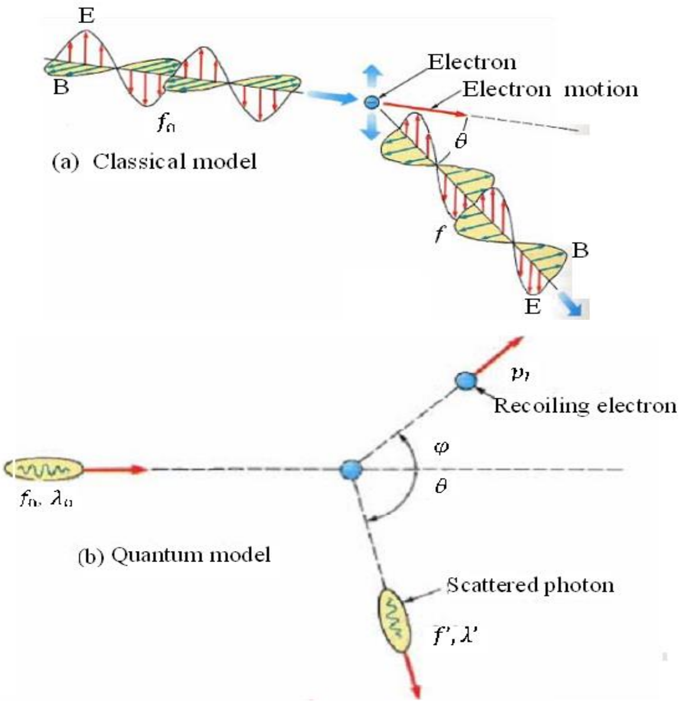

Figure 3.1 - X-ray scattering from an electron: (a) the classical model, (b) the quantum model

A schematic diagram of the apparatus used by Compton is shown in figure 3.2a. In his original experiment Compton measured the dependence of scattered x-ray intensity on wavelength at three scattering angles. The wavelength was measured with a rotating crystal spectrometer using carbon as the target, and the intensity was determined by an ionization chamber that generated a current proportional to the x-ray intensity.

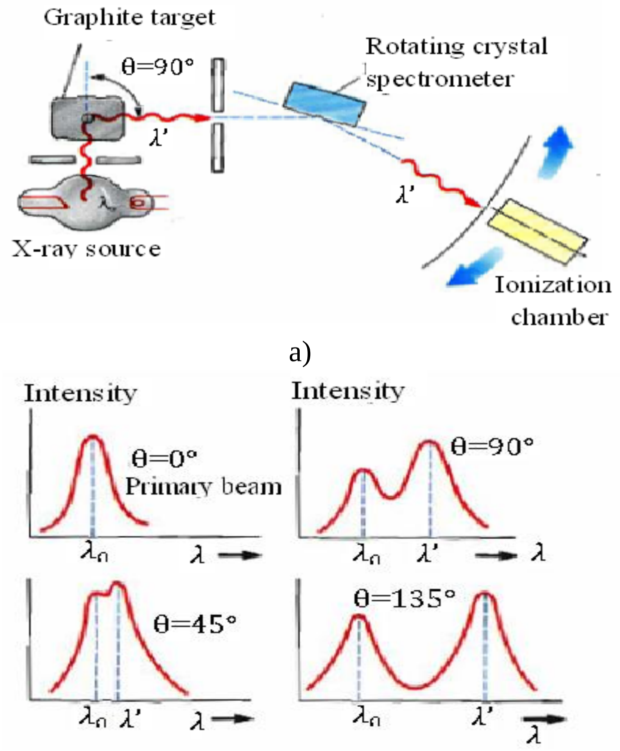

Figure 3.2 – a) Schematic diagram of Compton's apparatus. b) Scattered intensity versus wavelength of Compton scattering at $\theta = 0^{\circ}$, $\theta = 45^{\circ}$, $\theta = 90^{\circ}$, $\theta = 135^{\circ}$.

The incident beam consisted of monochromatic x-rays of wavelength $\lambda_0 = 0.071\,\text{nm}$. The experimental intensity versus wavelength plots observed by Compton for scattering angles of $0^{\circ}, 45^{\circ}, 90^{\circ}, 135^{\circ}$ are shown in figure 3.2b. They show two peaks, one at λ_0 and a shifted peak at λ'. The peak at λ_0 is caused by x-rays scattered from electrons that are tightly bound to the target atoms, while the shifted peak is caused by scattering of x-rays from free electrons in the target. In his analysis, Compton predicted that the shifted peak should depend on scattering angle θ as

$$\Delta\lambda = \lambda' - \lambda_0 \frac{h}{m_e c}(1 - \cos\theta) \qquad (3.1)$$

In this expression, known as the Compton shift equation, *m* is the mass of the electron, $h / m_e c$ is called the Compton wavelength of the electron; and has a currently accepted value of

$$\lambda_c = 2\pi\hbar/(m_e c) = 0.024 * 10^{-10}\,\text{m}.$$

Compton's measurements were in excellent agreement with the predictions of equation 3.1. It is fair to say that these were the first experimental results to convince most physicists of the fundamental validity of the quantum theory.

Derivation of the Compton shift equation

We can derive the Compton shift expression, equation 3.1, by assuming that the photon exhibits particle-like behavior and collides elastically with a free electron initially at rest, as in figure 3.3. In this model, the photon is treated as a particle of energy $E = hf = hc/\lambda$, with a rest mass of zero. In the scattering process, the total energy momentum of the system must be conserved.

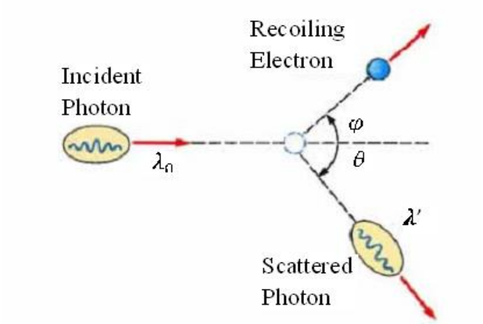

Figure 3.3. Diagram representing Compton scattering of a photon by an electron

The scattered photon has less energy (or longer wavelength) than the incident photon. Applying conservation of energy to this process gives

$$\frac{hc}{\lambda_0} = \frac{hc}{\lambda^{'}} + K_e \tag{3.2}$$

where hc/λ_0 is the energy of the incident photon, $hc/\lambda^{'}$ is the energy of the scattered photon, and K_e is the kinetic energy of the recoiling electron. Since the electron may recoil at speeds comparable to the speed of light, we must use the relativistic expression for K_e, given by $K_e = \gamma mc^2 - mc^2$.

Therefore,

$$\frac{hc}{\lambda_0} = \frac{hc}{\lambda^{'}} + \gamma mc^2 - mc^2 \tag{3.3}$$

where $\gamma = 1/\sqrt{1 - v^2/c^2}$.

Next, we can apply the law of conservation of momentum to this collision, noting that both the x and y components of momentum are conserved. Since the momentum of a photon has a magnitude given by $p = E/c = h/\lambda$, and since the relativistic expression for the momentum of the recoiling electron is $p_e = \gamma mv$, we obtain the following expressions for the x and y components of linear momentum:

x component:

$$\frac{h}{\lambda_0} = \frac{h}{\lambda^{'}}\cos\theta + \gamma mv\cos\phi \tag{3.4}$$

y component:

$$0 = \frac{h}{\lambda_0} = \frac{h}{\lambda^{'}}\sin\theta - \gamma mv\sin\phi \tag{3.5}$$

By eliminating v and ϕ from equation 3.2 to 3.4, we obtain a single expression that relates the remaining three variables ($\lambda^{'}, \lambda_0$ and θ). After some algebra, one obtains the Compton shift equation:

$$\Delta\lambda = \lambda^{'} - \lambda_0 = \frac{h}{mc'}(1 - \cos\theta) \tag{3.6}$$

We have seen that experiments such as blackbody radiation, the photoelectric effect, and Compton scattering can be explained using the photon picture of light, but not with the wave picture. However, it is important to realize that experiments such as diffraction and interference all need the wave picture, as a photon (particle) picture fails in these cases. Both pictures are needed in different circumstances; one says that light exhibits a wave-particle duality: Light has a dual nature; in some cases it behaves as a wave, and in other cases it behaves as a photon. This wave-particle duality is the basis of the quantum theory of light, and has some profound physical and philosophical implications which are still being debated today.

de Broglie waves

In 1924 a young physicist, de Broglie, speculated that nature did not single out light as being the only matter which exhibits a wave-particle duality. He proposed that ordinary "particles" such as electrons, protons, or bowling balls could also exhibit wave characteristics in certain circumstances.

The de Broglie equations relate the wavelength λ and frequency *f* to the momentum *p* and the energy *E*, respectively, as

$$\lambda = h / p \text{ and } f = E / h$$

where h is Planck's constant. The two equations are also written as :

$$p = \hbar k \text{ and } E = h f$$

Quantitatively, he associated a wavelength λ to a particle of mass *m* moving at speed v

$$\lambda = h / (mv)$$

Because the momentum of such a particle is $p = mv$

Experimental confirmation

In 1928 at Bell Labs, Clinton Davisson and Germer observed slow-moving electrons at a crystalline nickel target. The angular dependence of the reflected electron intensity was measured, and was determined to have the same diffraction pattern as those predicted by Bragg for x-rays. Before the acceptance of the de Broglie hypothesis, diffraction was a property that was to be only exhibited by waves.

Therefore, the presence of any diffraction effects by matter demonstrated the wave-like nature of matter. When the de Broglie wavelength was inserted into the Bragg condition, the observed diffraction pattern was obtained, it became experimental confirmation of the de Broglie hypothesis for electrons.

This was a pivotal result in the development of quantum mechanics. Just as Arthur Compton demonstrated the particle nature of light, the Davisson-Germer experiment showed the wave –nature of matter and completed the theory of wave-particle duality. For physicists this idea was important, because it means that not only any particle can exhibit wave characteristics, but that one can use wave equations to describe phenomena in matter if one uses the de Broglie wavelength.

Since the original Davisson-Germer experiment for electrons, the de Broglie hypothesis has been confirmed for other elementary particles. The wavelength of a thermalized electron in a non-metal at room temperature is about 8nm.

From meaning of the psi- function it follows that the quantum mechanics has a statistical character. It doesn't allow define a particle precise position in space. So, with reference to a microparticle concept of a trajectory loses sense. With the help of psi- function the probability of the particle finding in various points of space can be evaluated.

Problems

1.Calculate the de Broglie wavelength of an electron and a proton moving with a kinetic energy 1 keV. For what values of the kinetic energy their wavelength will be equal to $1\,\mathring{A}$.

2. With increasing of the electron energy on 200 eV, its de Broglie wavelength is changed twice. Find the original wavelength of the electron.

3. What additional energy must be imparted to an electron with a momentum 15 keV/s so that its wavelength would become equal to 0.5 $\mathring{A}$?

4. Proton with a wavelength $\lambda = 0.017\,\mathring{A}$ elastically scattered at an angle 90° on the originally stationary particle, whose mass at $n = 4$ times larger the proton mass. Determine the wavelength of the scattered proton.

5. Find the kinetic energy, at which the de Broglie wavelength of the electron is equal to its Compton wavelength.

Literatures

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